Check-Up

10

The commutative laws

This check-up and the following two are designed to make explicit some fundamental laws of arithmetic. Whatever the numbers a and b,

a) is it always true, sometimes true, or never true that $a + b = b + a$?

b) is it always true, sometimes true, or never true that $a - b = b - a$?

c) is it always true, sometimes true, or never true that $ab = ba$?

d) is it always true, sometimes true, or never true that $a \div b = b \div a$?

Answers to check-up 10

a) Always true. b) Sometimes true. c) Always true. d) Sometimes true.

Discussion and explanation of check-up 10

The statements in (a) and (c) are known as the *commutative laws* of addition and multiplication. Remember that ab in algebraic notation is a shorthand for 'a multiplied by b'. The generalisations that $a + b = b + a$ and $ab = ba$, whatever numbers are chosen for a and b, state simply that it does not matter in which order you add two numbers together and it does not matter in which order you multiply them together.

For example, using the commutative law of addition we could change 3 + 8999 into 8999 + 3. This is probably what most of us would do mentally, because it's easier to start at 8999 and count on by 3 (to get 9002) than to start at 3 and count on by 8999! The commutative law of multiplication allows us, for example, to change '5 sevens' into '7 fives'. Most people find it is easier to visualise counting in fives than in sevens. If the desks in your classroom were arranged in 5 rows of 7, you could also think of them as 7 rows of 5.

Here's a useful application of the commutative law of multiplication using percentages. Say you have to calculate 28% of £25. Well, because multiplication is commutative, you would get the same result if you calculate 25% of £28, which is very easy to do mentally ($\frac{1}{4}$ of £28 = £7).

Mathematicians say that addition and multiplication are *commutative*. Subtraction and division are not. With subtraction, it matters in what order the numbers are written down. '8 – 3' (= 5) is not the same thing as '3 – 8' (= -5). Think of subtraction here as counting back along a number line: '3 - 8' would take you down to –5. (This is like the temperature starting at 3ºC, then falling by 8º.) With division, '12 ÷ 3' and '3 ÷ 12' are certainly not interchangeable: $12 \div 3 = 4$, but $3 \div 12 = \frac{1}{4}$ or 0.25. So, if you have a difficult division calculation to do, such as finding the price per gram of 1450 grams of some substance costing £27.99, you have to think carefully about the order in which you enter the numbers on the calculator. In this example, to find the price per gram, you divide the price by the weight: $27.99 \div 1450 = 0.0193034$ = 0.02 approximately, giving £0.02 or 2p per gram.

The only occasions when $a - b = b - a$ and $a \div b = b \div a$ are true are when a and b are the same number. For example, $7 - 7 = 7 - 7$ and $42 \div 42 = 42 \div 42$, rather obviously, which is why the answers to (b) and (d) are 'sometimes true'. (There is one further exception in the case of division. We have to exclude the case where a and b are both zero. Any division by zero is meaningless. Try $0 \div 0$ on your calculator: all you will get will be an indication that you have made an error.)

Summary of key ideas

- Addition and multiplication are commutative, which means that it does not matter in what order you add them together or multiply them together.
- In algebraic form we say that, for any numbers a and b, $a + b = b + a$ and $ab = ba$.
- Subtraction and division are not commutative: $a - b$ is not equal to $b - a$ and $a \div b$ is not equal to $b \div a$ (except when a and b are the same number).

Further practice

10.1 Use the commutativity of multiplication to help calculate mentally: (a) 48% of £75 and (b) 35% of £60.

10.2 On a school skiing holiday, the coach takes 14.5 hours for the 895 km journey. What would you enter on a calculator to find the average speed in kilometres per hour: 14.5 ÷ 895 or 895 ÷ 14.5?

10.3 True or false?

a) 28   0 = 28   b) 0   28 = 0   c) 28 ÷ 0 = 28   d) 0 ÷ 28 = 0